Disruption of quantum coherence upon a change in spatial topology in quantum gravity

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Quantum coherence may be disrupted when closed universes of Planckian size split off from a plane space. The observation of $K^0 - \bar{K}^0$ oscillations imposes a restriction on the amplitude for such processes. This amplitude must be strongly suppressed in comparison with the dimensional estimate $M_{\rm Pl}^4$.

The possibility of a change in the topology of a three-dimensional space because of quantum-gravity effects has been discussed by many investigators, beginning with Wheeler.[1] In this letter we wish to point out that a change in the topology of the space may lead to a significant disruption of quantum coherence. We derive a limitation on the rate of processes involving a change in topology. This limitation follows from the existence of oscillations in the system of K^0 mesons.

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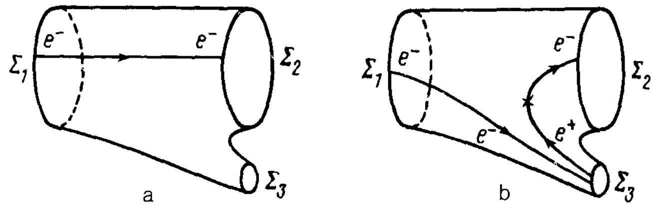

FIG. 1

A disruption of quantum coherence arises in processes in which a closed universe Σ_3 splits off from a large universe Σ_1 (Fig. 1), so that the final three-dimensional manifold has several connection components (Σ_2 and Σ_3 in Fig. 1). Dimensional considerations suggest that the size of the component Σ_3 would be on the order of the Planckian size. The splitting off of a closed universe does not contradict the energy, momentum, electric-charge, etc., conservation laws. The reasons for the loss of coherence are illustrated in Fig. 1. The state of a particle (e.g., an electron), $|\Psi_{\rm in}\rangle = |\mathbf{p},e^-\rangle$, is converted as a result of formation of a small universe into a state

$$|\Psi_{out}\rangle = A\{|\mathbf{p},e^-\rangle \otimes |0\rangle + \epsilon_{\mathbf{p}}|\mathbf{p},e^-\rangle \otimes |e^+e^-\rangle\}, \tag{1}$$

where $\epsilon_{\mathbf{p}}$ is the amplitude for the production of an e^+e^- pair, and A is the amplitude of the topological transition. From the standpoint of an observer living in the large universe and having no information on the state of the small universe, the pure state $\int d\mathbf{p} f(\mathbf{p})|\mathbf{p},e^-\rangle$ becomes a mixed state described by the density matrix

$$\begin{aligned}\rho_{out} &= \int d\mathbf{p}d\mathbf{q} f^*(\mathbf{p}) f(\mathbf{q}) \left\{1 - \frac{1}{2}(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{q}})^2 A^2\right\} |\mathbf{q}\rangle\langle\mathbf{p}|, \\ 1 - \mathrm{Sp}(\rho^2_{out}) &= \int d\mathbf{p}\, d\mathbf{q}\, |f(\mathbf{p})|^2 |f(\mathbf{q})|^2 (\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{q}})^2 A^2 > 0.\end{aligned} \tag{2}$$

It can be seen from this expression that the quantity $|\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{q}}|A$ is a quantitative measure of the loss of coherence. The possibility of a disruption of quantum coherence has been studied by Hawking[2] in connection with the evaportion of black holes and also by Page[3] and Hawking[4] in a discussion of the wave function of the universe.[1)]

Let us assume that the change in topology is a tunneling process which can be described by a Euclidean formalism. The manifold shown in Fig. 1 thus has a Euclidean signature.[2)] A calculation of the amplitude $\epsilon_{\mathbf{p}}$ is equivalent to the problem of the production of particles during a tunneling. In this case the behavior of the material fields is described by a Euclidean analog of the Schrödinger equation.[7] Solving this equation for an O(4)-invariant model metric in the theories of massive scalar and spinor fields, with a single-particle state used as a given initial state, we find expression (1) with

$$\epsilon_{\mathbf{p}} = \frac{1}{4\pi} l^2 \frac{T}{\omega_{\mathbf{p}}} (1 + O(m^2 l^2)) \qquad (\text{spin } 0), \tag{3a}$$

$$\epsilon_{\mathbf{p}} = \frac{1}{4\pi} l^3 \frac{mT}{\omega_{\mathbf{p}}} (1 + O(m^2 l^2)) \qquad (\text{spin } 1/2). \tag{3b}$$

Here $\omega_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$; l is the size of the small universe; and T is the normalization time (the rate of the processes in which we are interested is proportional to $\epsilon_{\mathbf{p}}/T$).

A loss of quantum coherence should lead to the disappearance of oscillations in the system of neutral K mesons. The time scale of this loss of coherence, τ, can be estimated from (2) and (3b). Specifically, using $l \sim M_{\text{pl}}^{-1}$ for this estimate, we find $\tau^{-1} \sim AM_{\text{pl}}^{-3} [(1/m_L) - (1/m_S)]m_q$, where m_L and m_S are the masses of the K_L and K_S mesons, and m_q is the mass of a light quark. Working from dimensional considerations, we find $A = aM_{\text{pl}}^4$ for the amplitude of the topological transition per unit volume per unit time, where a is a dimensionless coefficient. Comparing τ with the oscillation time $\Delta t_{\text{osc}} \sim 10^{-10}$ s, we find the estimate $a \lesssim 10^{-17}$. The amplitudes for transitions involving a change in topology are thus greatly suppressed in comparison with the natural value M_{pl}^4. Ths result may indicate that the gravitational interaction does not become strong even at a Planckian scale.

Processes involving a change in spatial topology may lead to some interesting effects, e.g., the appearance of a nonlocality and a nonconservation of global quantum numbers (the baryon number, the lepton number, etc.).

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[1)]This study was reported at the Fourth International Seminar on Quantum Gravity (Moscow, May 1987). At the Seminar, a similar mechanism for the loss of coherence was proposed in a paper by Hawking,[5] who treated the case of massless particles. Our results for massless particles agree with Hawking's results. In the case of fermions, however, the masses of the particles play a fundamental role (more on this below), so our basic conclusion that there is an effect and also the estimate of its magnitude in the case of fermions are qualitatively different from Hawking's conclusions.

[2)]An alternative approach using a manifold with a Lorentzian signature runs into serious difficulties.[6]

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